Cadet

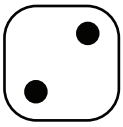
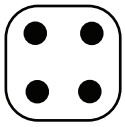
3 points

1. Lisa has four wooden digits. She can use them to form the number 2025. Which of the following numbers is the largest she can form with these digits?

(**A**) 2502 (**B**) 5202 (**C**) 5220 (**D**) 5502 (**E**) 5520

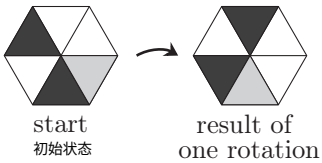
SOLUTION: To get the maximum number digits must be arranged from highest to lowest hence 5220.

2. Sandra rolls three dice and gets a total of 8. All three dice show a different number of dots. Which number of dots could Sandra not have rolled on any of her dice?

(**A**)  (**B**)  (**C**)  (**D**)  (**E**) 

SOLUTION: To get 8 as the sum of three distinct numbers 1- 6 , the only possibilities are 5+2+1 and 4+3+1. Hence 6 is the number of dots she could not have rolled.

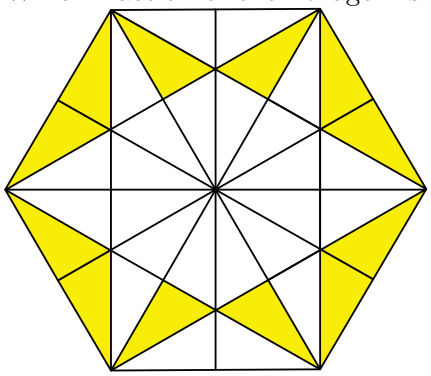
3. Isabelle rotates the hexagonal sheet of paper, as shown. Each rotation turns the hexagon through the same angle in the same direction. The figure shows the result of one rotation.
Which of these numbers of rotations would leave the sheet looking the same as it did at the start?



(**A**) 7 (**B**) 8 (**C**) 9 (**D**) 10 (**E**) 12

SOLUTION: The sheet looks like in the beginning after a number of rotations that is divisible by 6. The only number in the answers divisible by 6 is 12.

4. The regular hexagon shown is divided into many triangles of equal area.
Which fraction of the hexagon is unshaded?



(**A**) $\frac{2}{3}$ (**B**) $\frac{3}{4}$ (**C**) $\frac{4}{5}$ (**D**) $\frac{5}{6}$ (**E**) $\frac{6}{7}$

SOLUTION: The regular hexagon is divided into 36 triangles of which 24 are unshaded. Then the required fraction is equal to $\frac{24}{36} = \frac{2}{3}$.

5. How many lots of 12 minutes are there in 12 hours?

(**A**) 60 (**B**) 24 (**C**) 12 (**D**) 10 (**E**) 6

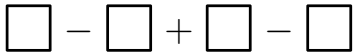
SOLUTION: There are 5 lots of 12 minutes in 1 hour. Hence there are $5 \times 12 = 60$ lots of 12 minutes in 12 hours.

6. Daniel is 5 years old. His brother Dominic is 6 years older.
What will the sum of their ages be in 7 years' time?

(**A**) 26 (**B**) 27 (**C**) 28 (**D**) 29 (**E**) 30

SOLUTION: In 7 years' time, Daniel's age will be $5 + 7 = 12$. In 7 years' time, Dominic's age will be $(5 + 6) + 7 = 18$. Therefore, the sum of their ages will be $12 + 18 = 30$.

7. Ohad wants to write the four digits 2, 0, 2 and 5 in the four boxes of the calculation shown.



What is the smallest result that Ohad could get?

(**A**) -7 (**B**) -6 (**C**) -5 (**D**) -4 (**E**) -3

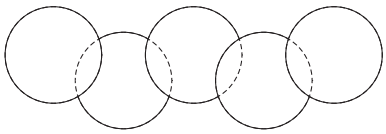
SOLUTION: Since he is effectively adding two of the digits and then subtracting the other two, Ohad will get the smallest result from his calculation when the two digits he places after the subtraction signs in the calculation are as large as possible. Hence he should subtract 5 and 2 to get the smallest result. One such calculation where this is the case is $2-2+0-5=-5$. Therefore the smallest answer Ohad could get is -5.

8. There are ten more truth-tellers than liars in a room. Everyone in the room was asked, "Are you a truth-teller?" and everyone gave an answer. A total of 20 people answered, "Yes."
How many truth-tellers are in the room?

(**A**) 0 (**B**) 5 (**C**) 15 (**D**) 20 (**E**) 25

SOLUTION: Truth-tellers must answer "yes" and liars will also answer "yes" as they are not truth tellers. Therefore, the total number of people in the room is 20. Since there are 10 more truth-tellers than liars, there are 5 liars in the room and there are 15 truth-tellers.

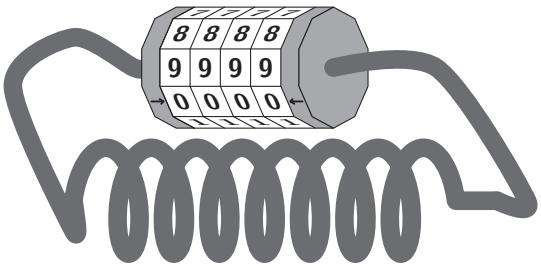
9. Five circles, each with an area of 8 cm^2, overlap to form the figure shown. The area of each section where two circles overlap is 1 cm^2. What is the total area covered by the figure?



(**A**) 32 cm^2 (**B**) 36 cm^2 (**C**) 38 cm^2 (**D**) 39 cm^2 (**E**) 42 cm^2

SOLUTION: There are four regions of overlap in the diagram with total area $4 \times 1 \text{ cm}^2 = 4 \text{ cm}^2$. Therefore the total area covered by the figure is $5 \times 8 \text{ cm}^2 - 4 \text{ cm}^2 = 36 \text{ cm}^2$.

10. The real combination for the bicycle lock shown in the picture is 0000. However, when someone looks at it from the side, they see 8888.
When Paul looks at the combination of his friend's lock from the side, he sees 8417.
What is the real combination of his friend's lock?

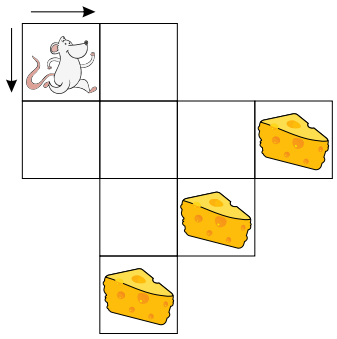


(**A**) 4037 (**B**) 4693 (**C**) 0639 (**D**) 0693 (**E**) 9603

SOLUTION: Someone looking from the side sees 8888 if the real combination is 0000, hence someone looking from the side has to add 2 to every digit to know the real combination. Therefore 0639 is the real combination from the lock of Paul's friend.

4 points

11. Matjaz the mouse wants to get to a piece of cheese. He can only move horizontally or vertically between any two cells in the directions shown by the arrows.
How many different routes can Matjaz take to reach a piece of cheese?



(**A**) 3 (**B**) 5
(**C**) 8 (**D**) 10
(**E**) 11

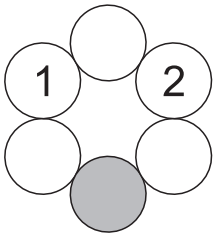
SOLUTION: Pivotal square is the one diagonally opposite to the mouse -2 ways to reach it. This gives 2 ways to reach the far right cheese and 2 ways to reach the bottom cheese. The cheese between these two has four ways (2×2) hence 8 is the answer.

12. There are five hurdles in a 60 m hurdles race. The first hurdle is after 12 m. The gap between any two consecutive hurdles is 8 m.
How far is the last hurdle from the finish?

(**A**) 16 m (**B**) 14 m (**C**) 12 m (**D**) 10 m (**E**) 8 m

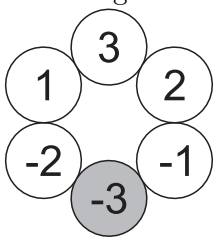
SOLUTION: The 5th hurdle will be after 12 m$+8$ m$\times 4 = 12$ m$+32$ m $= 44$ m. This is 60 m-44 m $=$ 16 m from the finish.

13. Edgar wants to write a number in each circle in the diagram. He wants each number to be equal to the sum of the numbers in the two adjacent circles. He has already written two numbers, as shown. What number should he write in the grey circle?

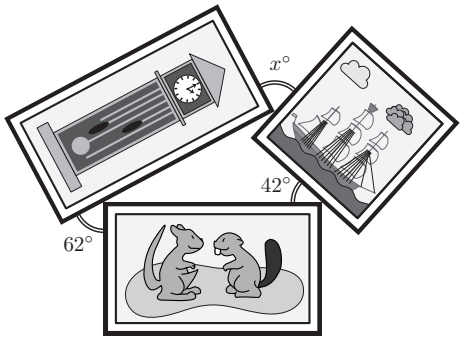


(**A**) 2 (**B**) -1 (**C**) -2
(**D**) -3 (**E**) -5

Solution: It is clear that in the top circle it must be 3. Since the sum of 3 and the open circle at the left must be 1 , there must be -2 in the left circle. At the same time the sum of 3 and the circle on the right must be 2, so in the right circle must be -1. Hence in the bottom circle there must be -3.

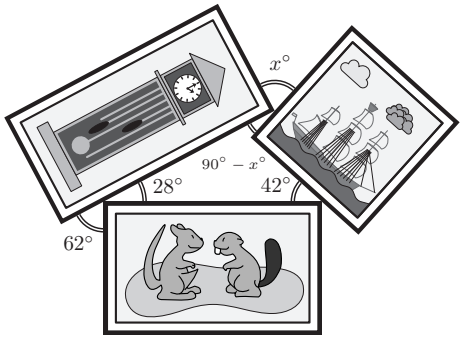


14. Louise places three rectangular pictures in the way shown.
What is the value of x?



(**A**) 64 (**B**) 70 (**C**) 72 (**D**) 76 (**E**) 80

Solution: $90 - 62 = 28$; $28 + 42 + (90 - x) = 180 - 90$; $x = 70$



15. Werner is on a treadmill in the gym. He keeps looking at two stopwatches. The first shows the time elapsed since he started his session and the second the time remaining until the end of his session.

14:58	21:32

At some point the two stopwatches show the same reading.
What do they show at that point?

(**A**) 17:50 (**B**) 18:00 (**C**) 18:12
(**D**) 18:15 (**E**) 18:20

SOLUTION: There is a difference of six minutes and 34 seconds between one stopwatch and the other. They will coincide when 3 minutes and 17 seconds have elapsed for both. If we add them to the first stopwatch, it will read 18:15 and if we subtract them from the second stopwatch, it will also read 18:15.

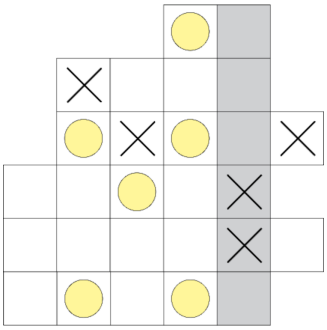
16. Julia wants to fill each $\square$ with a different prime number less than 20 so that the value of A is an integer.
What is the maximum value of A?

$$A = \frac{\square + \square + \square + \square + \square + \square + \square}{\square}$$

(**A**) 20 (**B**) 14 (**C**) 10 (**D**) 8 (**E**) 6

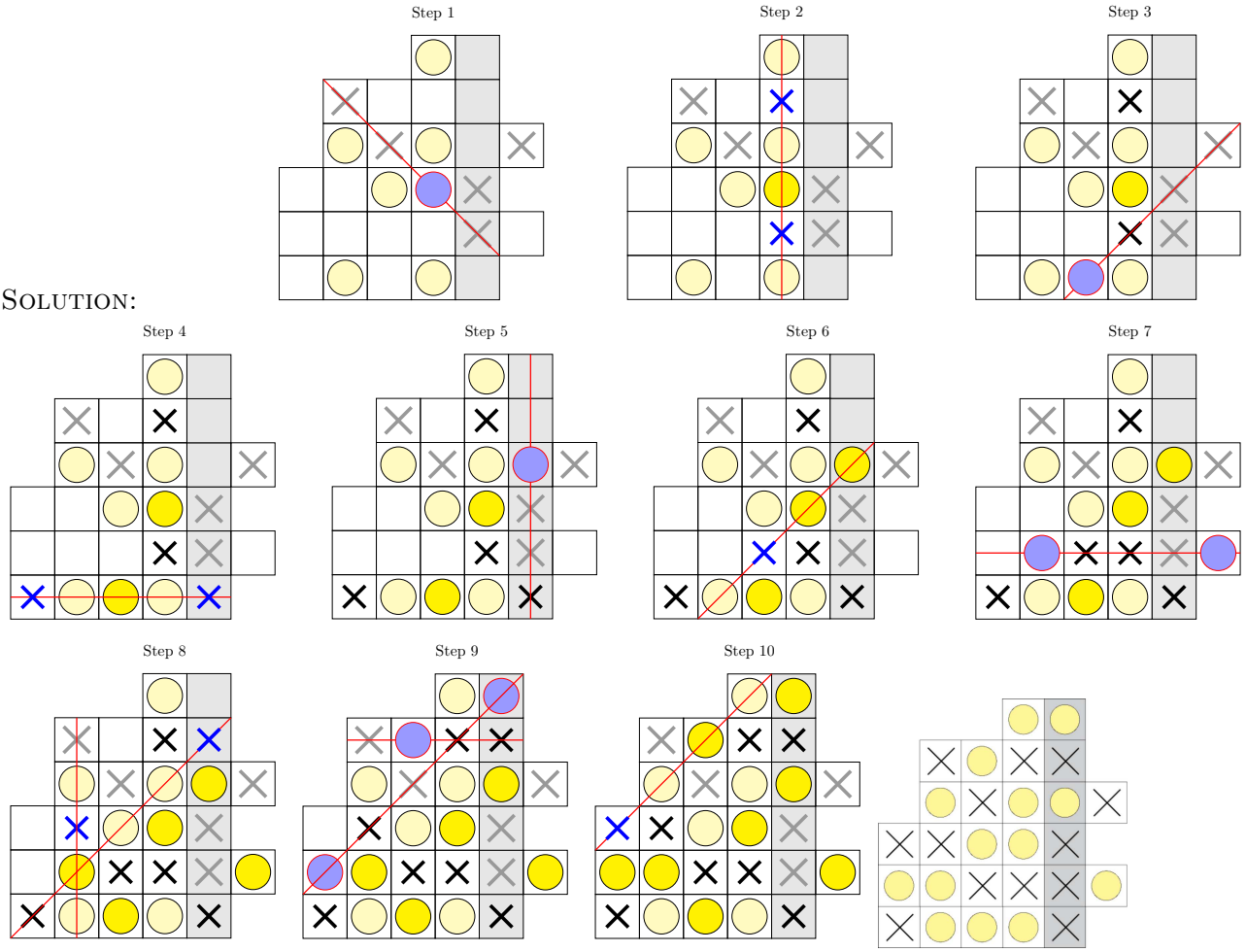
SOLUTION: There are only eight prime numbers less than 20, namely 2, 3, 5, 7, 11, 13, 17 and 19. It follows that the sum of the numerator and denominator is $2+3+5+7+11+13+17+19 = 77$. Since A is an integer, this implies the denominator is 7 or 11. When the denominator is 7, the value of A is (77 - 7)/7 = 10. When the denominator is 11, the value of A is (77 - 11)/11 = 6 Thus, the maximum value of A is 10.

17. Morten wants to fill in the cells on the diagram shown so that each cell contains either a cross or a circle. He also wants to ensure there is no line of four consecutive identical symbols in any column, row or diagonal. When has completed the diagram, what will the column coloured grey contain?

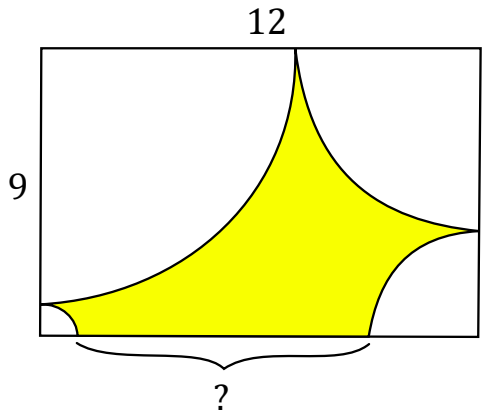


(**A**) 3 circles and 3 crosses
(**B**) 2 circles and 4 crosses
(**C**) 4 circles and 2 crosses
(**D**) 5 circles and a cross
(**E**) a circle and 5 crosses

SOLUTION:



18. Peter has drawn a quarter circle with centre at each corner of a flag with dimensions 12 cm by 9 cm and coloured the region formed, as shown.
What is the length indicated by the question mark?



(**A**) 5 cm (**B**) 6 cm (**C**) 7 cm (**D**) 8 cm (**E**) 9 cm

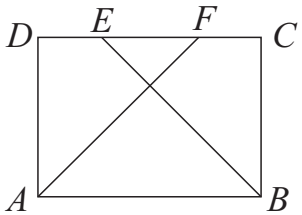
SOLUTION: The flag is a rectangle, hence, its opposite sides are equal. Looking at the two smallest ones, we realize that the radii of the four circles add up to $2 \cdot 9$ cm $= 18$ cm. The other two sides, that is the four radius and the missing length, add up to $2 \cdot 12\ cm = 24$ cm. Therefore, the answer is 24 cm $-$ 18 cm $= 6$ cm.

19. Sanja has two bowls of numbered balls. Bowl X contains seven balls numbered 1, 2, 6, 7, 10, 11 and 12. Bowl Y contains five balls numbered 3, 4, 5, 8, and 9. Which ball should Sanja transfer from Bowl X to Bowl Y to increase the average number on the balls in each bowl?

(**A**) 6 (**B**) 7 (**C**) 10 (**D**) 11 (**E**) 12

SOLUTION: We need to transfer a ball with a number larger than the average of the numbers in one bowl, yet smaller than the average of numbers from the second bowl. The average of first is 7, while in bowl 2 the average is 5.8. Then, the number we can transfer is 6.

20. In the rectangle $ABCD$, the points E and F are marked on side DC, as shown, so that $\angle EBA = \angle DFA = 45°$ and $AB + EF = 20$ cm.
What is the length of BC?



(**A**) 4 cm (**B**) 6 cm (**C**) 8 cm (**D**) 10 cm (**E**) 12 cm

SOLUTION: IF $AB = a$ and $BC = b$, then $EF = 2b - a$. Hence $AB + EF = 2b$ and then $b = BC =$ 20 cm : $2 = 10$ cm.

5 points

21. In the six-digit integer $PAPAYA$, different letters stand for different digits and the same letter always represents the same digit. Also $Y = P + P = A + A + A$.
What is the value of $P \times A + P + A \times Y \times A$?

(**A**) 33 (**B**) 39 (**C**) 66 (**D**) 132 (**E**) 252

SOLUTION: Since $Y = P + P = A + A + A$, we know that Y is both a multiple of 2 and a multiple of 3 and hence is a multiple of 6. Therefore, since Y is a single digit, $Y = 6$ and hence $P = 3$ and $A = 2$. Therefore $P \times A + P + A \times Y \times A = 3 \times 2 + 3 + 2 \times 6 \times 2 = 33$.

22. During two sessions of football training, Paul shoots a total of 17 times at a target. He hits with 60% of the shots he shoots in the first session. He hits with 75% of the shots he shoots in the first session.
How many times did he hit the target in the second session?

(**A**) 3 (**B**) 5 (**C**) 6 (**D**) 8 (**E**) 10

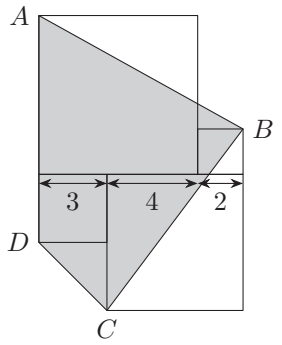
SOLUTION: Since $60\% = \dfrac{60}{100} = \dfrac{3}{5}$ of the shots in the first session has to be a whole number, this number has to be a multiple of 5. Hence, he shot 0, 5, 10 or 15 times in the first session. This means he shot $17 - 0 = 17$, $17 - 5 = 12$, $17 - 10 = 7$ or $17 - 15 = 2$ times in the second session. Of this number $75\% = \dfrac{3}{4}$ has to be a whole number, so this number has to be a multiple of 4. Therefore he made 12 shots in the second session and 5 shots in the first session, of which he hit $\dfrac{3}{5} \times 5 = 3$.

23. Anurag always leaves for school at 8:00 a.m. His school is 1 km away. When he walks, his speed is 4 km/h. When he cycles, his speed is 15 km/h. He is 5 minutes early when he walks. How many minutes early is he when he cycles?

(**A**) 12 (**B**) 13 (**C**) 14 (**D**) 15 (**E**) 16

SOLUTION: The time it takes Anurag to walk 1 km is 15 minutes, and the time it takes him to cycle 1 km is 4 minutes. Therefore, he saves 11 more minutes. Since he is 5 minutes early when walking, he is a total of 16 minutes early when he cycles.

24. Ria draws four squares side by side, as shown.
What is the area of the shaded quadrilateral?



(**A**) 54
(**B**) 60
(**C**) 66
(**D**) 72
(**E**) 80

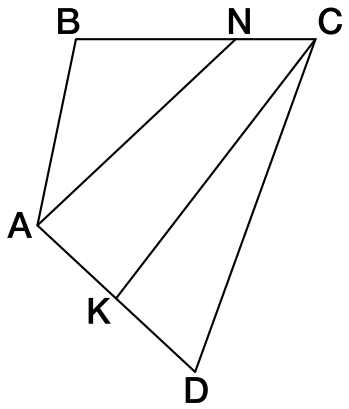
SOLUTION: By extending the lines, the resulting shape is a rectangle whose dimensions in cm are 13 and 9. If we subtract the areas of the three right triangles at it's corners from the area of the rectangle, then the required area will be equal to $13\text{ cm}\times 9\text{ cm}-\frac{1}{2}\times 9\text{ cm}\times 5\text{ cm}-\frac{1}{2}\times 3\text{ cm}\times 3\text{ cm}-\frac{1}{2}\times 6\text{ cm}\times 8\text{ cm} = 117\text{ cm}^2 - 22{,}5\text{ cm}^2 - 4{,}5\text{ cm}^2 - 24\text{ cm}^2 = 66\text{ cm}^2$.

25. The letters p, q, r, s and t represent five consecutive positive integers, though not necessarily in that order. The sum of p and q is 69 and the sum of s and t is 72. What is the value of r?

(**A**) 29 (**B**) 31 (**C**) 34 (**D**) 37 (**E**) 39

SOLUTION: Since the five integers are consecutive, the largest possible difference between them is four. The possible pairs of integers differing by four or less that add to 69 are 34 and 35 or 33 and 36. Similarly, the possible pairs of integers differing by four or less that add to 72 are 35 and 37 or 34 and 38.
If the pair adding to 69 were 34 and 35, neither of the pairs adding to 72 could be made without repeating an integer. Therefore, the pair adding to 69 is 33 and 36. Now, the pair of integers adding to 72 will be 35 and 37 or we would have a difference greater than four between two of the integers. Therefore four of the integers are 33, 36, 35 and 37 and hence the value of the missing integer, r, is 34.

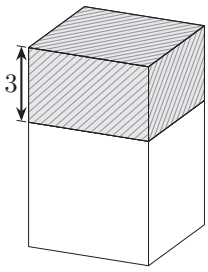
26. In the quadrilateral $ABCD$, the points N and K are marked on sides BC and AD respectively so that $BN = 2NC$ and $AK = KD$. The area of triangle CKD is 3, and the area of triangle ABN is 4.
What is the area of quadrilateral $ABCD$?



(**A**) 11 (**B**) 12 (**C**) 13 (**D**) 14 (**E**) 15

Solution: Draw AC. Since $2NC = BN$ the area of triangle $ANC = \frac{1}{2}\ ABN = 2$. The $S_{ACK} = S_{CKD}$ because CK is the median. Therefore $S_{ABCD} = S_{ABN} + S_{ANC} + S_{ACK} + S_{CKD} = 4+2+3+3 = 12$.

27. When the height of a cuboid is reduced by 3 cm, its surface area is reduced by 60 cm^2. The resulting shape is a cube.
What is the volume of the original cuboid, in cm^3?



(**A**) 75
(**B**) 125
(**C**) 150
(**D**) 200
(**E**) 225

Solution: Since the resulting shape when the height of the cuboid is reduced by 3 cm is a cube, the base of the cuboid is a square. Let the side-length of this square be x cm. The reduction in surface area from the cuboid to the cube comes from four equal rectangles, each of height 3 cm and length x cm. Therefore, we have $4 \times (3 \times x) = 60$ and hence $x = 5$. Therefore the original volume of the cuboid, in cm^3, was $5 \times 5 \times (5 + 3) = 5 \times 5 \times 8 = 200$.

28. Some birds, including Ha, Long, Nha and Trang, are perching on four parallel wires. There are 10 birds perched above Ha. There are 25 birds perched above Long. There are five birds perched below Nha. There are two birds perched below Trang. The number of birds perched above Trang is a multiple of the number of birds perched below her.
How many birds in total are perched on the four wires?

(**A**) 27 (**B**) 30 (**C**) 32 (**D**) 37 (**E**) 40

SOLUTION: Let wire 1 be at the top, followed below by wires 2, 3 and 4, respectively.

(i) Since there are more birds above Long than above Ha, Long must be positioned below Ha. Additionally, since there are still birds above Ha, Ha cannot occupy wire 1. By similar reasoning, Trang must be positioned on a wire below Nha and cannot be on wire 4.
(ii) With 10 birds above Ha and 25 above Long, there are at least 15 birds on Ha's wire or below it. If Nha were above Ha, those 15 birds would count toward the number below Nha. However, Nha has only 5 birds below her, so Nha cannot be above Ha. This implies that Nha must be on wire 2, sharing the wire with Ha, as it's the only position that satisfies both Ha's and Nha's conditions. Consequently, Trang must be on wire 3.
(iii) Long cannot be on the same wire as Trang, since that would imply Trang has 25 birds above her (equal to the number above Long). However, 25 is not an even multiple of the 2 birds below Trang, so Long must be positioned on wire 4.
Empty cells in the next tables show admissible positions at each step.

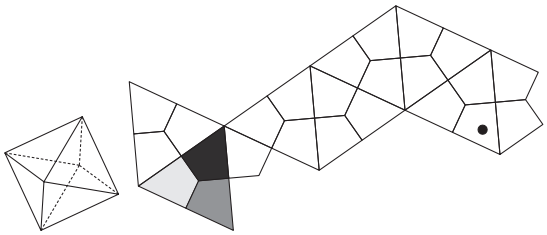
step (i)	Ha	Long	Trang	Nha
w1	X	X	X	
w2		X		
w3				X
w4	X		X	X

step (ii)	Ha	Long	Trang	Nha
w1	X	X	X	X
w2	O.K.	X	X	O.K.
w3	X		O.K.	X
w4	X		X	X

step (iii)	Ha	Long	Trang	Nha
w1	X	X	X	X
w2	O.K.	X	X	O.K.
w3	X	X	O.K.	X
w4	X	O.K.	X	X

There will be 2 birds in wire 4 and 25 birds in wires 1, 2, and 3. In total there will be 27 birds.
More precisely, there will be 2 birds in wire 4 (Long and another bird), 3 birds in wire 3 (Trang and 2 others), 12 birds in wire 2 (Ha, Nha and 10 others) and plus 10 birds in wire 1.

29. The right-hand figure below shows the adapted net of an octahedron. Each face of the octahedron is divided into three parts. The octahedron is coloured with the three colours black, dark grey and light grey in such a way so that the parts that come out of the same vertex or out of an opposite vertex are the same colour. Which colour could the part marked with a dot be coloured?

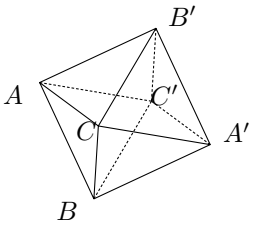


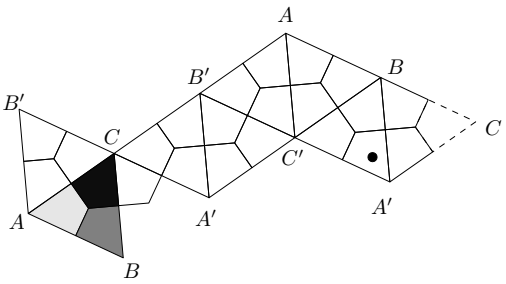
(**A**) Only black (**B**) Only dark grey (**C**) Only light grey

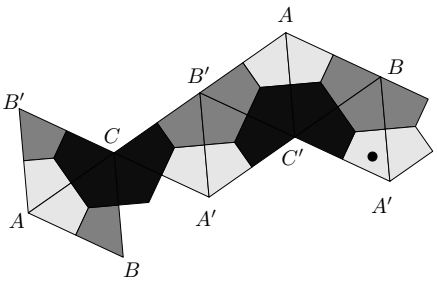
(**D**) Both black and dark grey are possible.

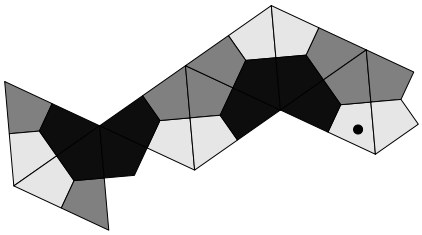
(**E**) Both black and light grey are possible.

SOLUTION: Let A' be the opposite vertex of vertex A, B' be the opposite vertex of vertex B and C' be the opposite vertex of vertex C.



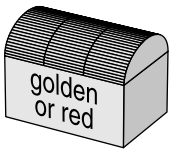


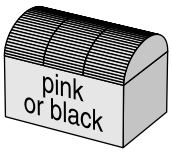


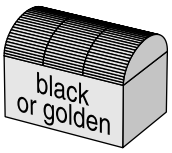


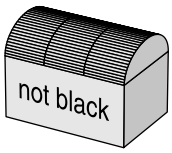
30. Adira keeps golden, red, black, pink and white pearls in five small boxes. Each box contains pearls of only one colour. The boxes are labeled as shown, and all the labels are true. Adira's friend Lilly wants to know which box contains the golden pearls. She may open exactly one of the five boxes to look inside.

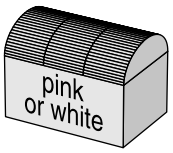
Which box should Lilly open to be certain which of the boxes contains the golden pearls?

(**A**) 

(**B**) 

(**C**) 

(**D**) 

(**E**) 

Solution: The answer is (D). If box D contains the golden pearls, we are done. If it contains red pearls, then box A contains the golden pearls, so we are done. If box D contains pink pearls, then box B contains black pearls and box C contains the golden pearls and we are done. Finally, if box D contains white pearls, then box E contains pink pearls, box B contains black pearls and box C contains the golden pearls and we are also done.

We also show that choosing one of the other boxes doesn't guarantee that Adira knows where the golden pearls are. If box A contains red pearls, then the other boxes could contain either black, golden, pink, white pearls (in the natural order) or pink, black, golden, white pearls, it is not clear where the golden pearls are. This argument works similarly for the other boxes.