

Benjamin

3 points

1 (Denmark). Holger fills the rest of the table with the numbers up to 40, following the system shown:

1	2	3	4	5	6	7	8
9	10	11	12				

Which of the pieces shown could he cut from the table?

- (A)

12
22 23
33
- (B)

12
20 21
28
- (C)

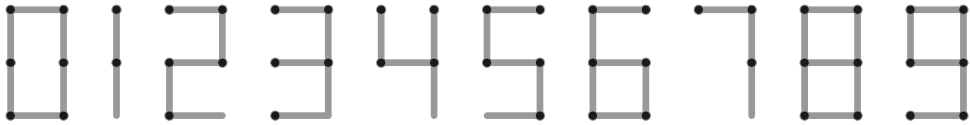
12
20 21
29
- (D)

12
21 22
30
- (E)

12
21 22
31

SOLUTION: A number has to be 8 greater than the number just above, hence C is the right answer.

2 (Austria). Matchsticks can be placed to build numbers, as shown. For example, to build the number 15, one needs 7 matchsticks, and one needs the same number of matchsticks to build the number 8.



What is the largest positive number that can be built with seven matchsticks?

- (A) 31
- (B) 51
- (C) 74
- (D) 711
- (E) 800

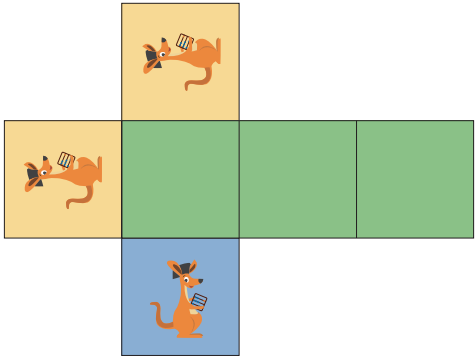
SOLUTION: The number 800 requires 19 matchsticks, but 711 can be written with exactly $3 + 2 + 2 = 7$ matchsticks.

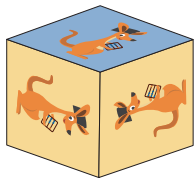
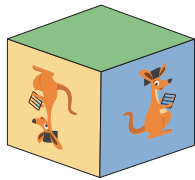
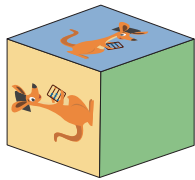
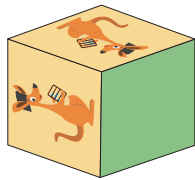
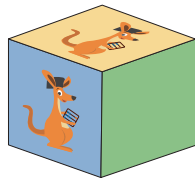
3 (Georgia). Which of the following shapes cannot be divided into two triangles by a single straight line?

- (A)
- (B)
- (C)
- (D)
- (E)

SOLUTION: (A) (B) (C) (D) (E)

4 (China). Rosalinde has a piece of paper, marked as shown, which she folds to form a cube.
Which of the following five cubes can she get from this paper?

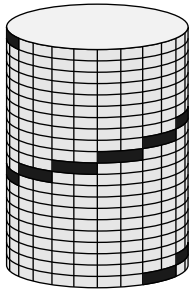


- (A)  (B)  (C)  (D)  (E) 

SOLUTION:

5 (Canada). Claude climbs from the bottom to the top of the cylindrical tower shown. The steps are all equal sized. Nine steps are visible. How many steps are not visible?

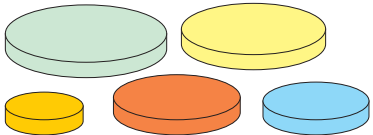
- (A) 9
(B) 10
(C) 11
(D) 12
(E) 13



SOLUTION: The idea is to disregard any thoughts of counting steps, and just realizing that you need one step per horizontal layer, hence 21 steps in total.

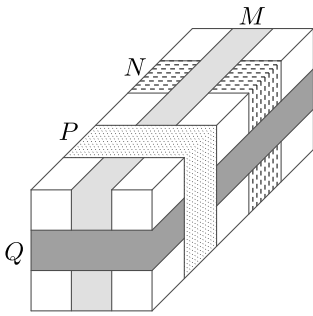
6 (Slovenia). Anna has five circular discs of different sizes. She wants to build a tower of four discs so that each disc in her tower is smaller than the disc immediately below it. How many different towers could Anna build?

- (A) 4 (B) 5 (C) 9 (D) 12 (E) 20



SOLUTION: To choose four discs out of five is the same as leaving one out. Hence there are five different ways to choose four discs. As soon as Anna has chosen the four discs, there is only one way to build the tower following the rules. If we numerate the discs 1 (smallest) to 5 (largest), the solutions are 2345, 1345, 1245, 1235, 1234.

7 (Catalonia). The picture shows a parcel around which four tapes labelled M , N , P and Q are placed.

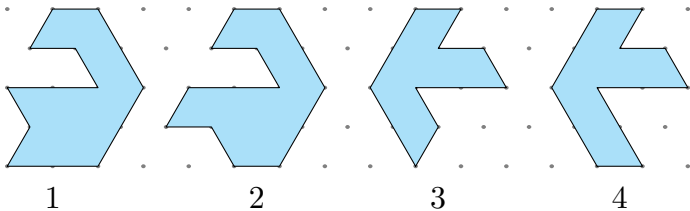


In what order, from first to last, were the tapes placed?

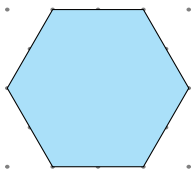
- (A) M, N, Q, P (B) N, M, P, Q (C) N, Q, M, P
(D) N, M, Q, P (E) Q, N, M, P

SOLUTION: P covers M, N and Q . Q covers M and N . M covers N . Therefore N is the first, followed by M , followed by Q and P is the last.

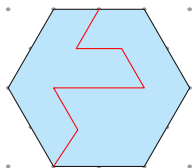
8 (Denmark). Alice has the four puzzle pieces shown.



Which pair can be combined to form this hexagon?

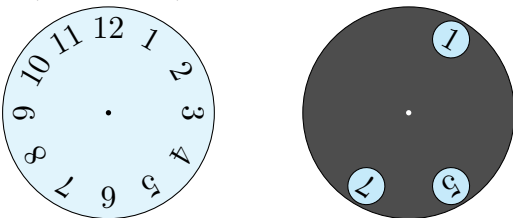


- (A) 1 and 2 (B) 1 and 3 (C) 2 and 3 (D) 2 and 4 (E) 1 and 4



SOLUTION:

9 (Denmark). The grey circle with three holes punched in it is placed on top of the clock-face.



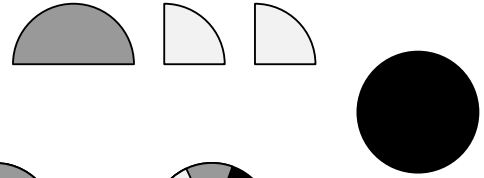
The grey circle is turned around its center. Which three numbers is it possible to see at the same time?

- (A) 2, 4 and 9 (B) 1, 5 and 10 (C) 4, 6 and 12 (D) 3, 6 and 9 (E) 5, 7 and 12

SOLUTION: The picture shows that one hole will show a time 4 hours after on of the other holes and 2 hours before the third hole. This is only true for C.

10 (Germany). Jonte glued the three pieces of paper shown onto the black circle on the right.

Which of the following patterns could he not obtain?



- (A) (B) (C) (D) (E)

SOLUTION: In the picture (C) the 'half-disk-piece' would span more than half of the disk and is therefore not possible.

4 points

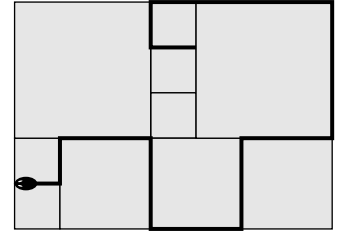
11 (Germany). Francesca wrote down three consecutive 2-digit numbers in their natural order, but instead of the digits she used symbols: $\square\Diamond$, $\heartsuit\Delta$, $\heartsuit\square$. Which number is next?

- (A) $\square\heartsuit$ (B) $\square\square$ (C) $\heartsuit\heartsuit$ (D) $\Diamond\square$ (E) $\heartsuit\Diamond$

SOLUTION: From the change of the first digit we conclude that $\Diamond = 9, \Delta = 0, \square = 1$. So the first number is 19, and we have $\heartsuit = 2$. So, the next number to write is 22, that means $\heartsuit\heartsuit$.

12 (Slovakia). The Potters have a patio which is tiled with square tiles of three different sizes. The smallest squares have a perimeter of 80 cm. A snake rests on the patio, as shown in the diagram. What is the length of the snake?

- (A) 380 cm (B) 400 cm
(C) 420 cm (D) 440 cm
(E) 1680 cm



SOLUTION: You get the side-length of the small tile by calculating $80 \text{ cm} : 4 = 20 \text{ cm}$
Then middle tile = 40 cm
And the big tile = 60 cm
Snake length = $5 \times 20 \text{ cm} + 5 \times 40 \text{ cm} + 2 \times 60 \text{ cm} = 420 \text{ cm}$

13 (Iran). When I look in a mirror, I can see the image of my digital clock standing on the table behind me, as shown.



What image will I see when I look in the mirror 30 minutes later?

- (A) (B) (C) (D) (E)

SOLUTION: The time in the beginning is 21:51.
After 30 minutes, the time is 22:21. Which is mirrored in answer D.

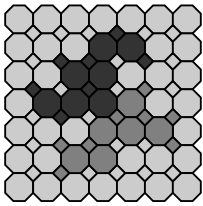
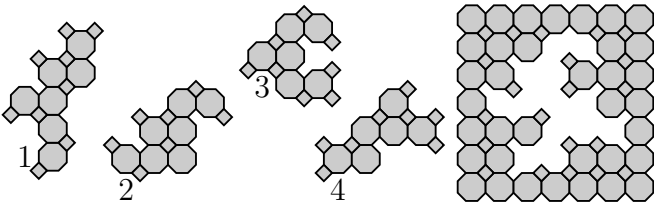
14 (Slovakia). Maria, Peter, Richard and Tina were playing football in the classroom and broke a window. When the principal asked who did it, she got the following responses: Maria: "It was Peter." Peter: "It was Richard." Richard: "It wasn't me." Tina: "It wasn't me." Only one child was telling the truth. Who broke the window?

- (A) Maria (B) Tina (C) Peter (D) Richard
(E) can't be determined with certainty

SOLUTION: If Richard tells the truth, then Peter lies and vice versa. Since only one child is telling the truth, we know it is either Peter or Richard, and hence Maria and Tina both lie That means Tina has broken the window.

15 (Germany). Which two tiles should be used to complete the puzzle?

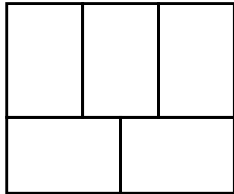
- (A) 1 and 2 (B) 1 and 4
(C) 2 and 3 (D) 2 and 4
(E) 3 and 4



SOLUTION:

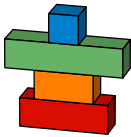
16 (United States). The diagram shows five rectangles. Lukas wants to colour the rectangles red, blue and yellow so that any two adjacent rectangles are coloured different colours. In how many different ways can he do this?

- (A) 3 (B) 4 (C) 5
(D) 6 (E) 7

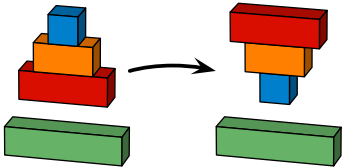


SOLUTION: Without loss of generality, put the color RED in the top left rectangle. Then, there are 2 options for the 2nd rectangle. Based on these 2 choices, the rest of the box can be colored since the bottom left will have to be the 3rd color, the bottom right would have to be RED, and the top right would have to be the same color as the bottom right. Thus for Blue and Yellow, there are similarly 2 options each so the number of possibilities is $3 \times 2 = 6$.

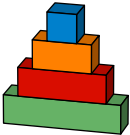
17 (Canada). Goran has four blocks, stacked as shown.



In a single move, Goran can take some, or all, of the blocks from the top of the stack and place them upside down, as shown.



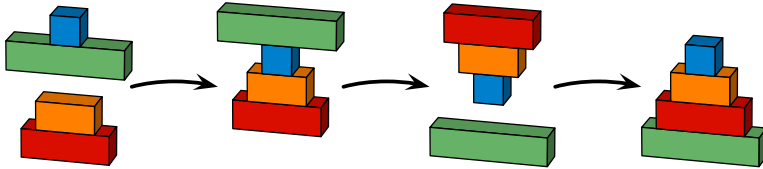
He wants the blocks to be stacked in this order:



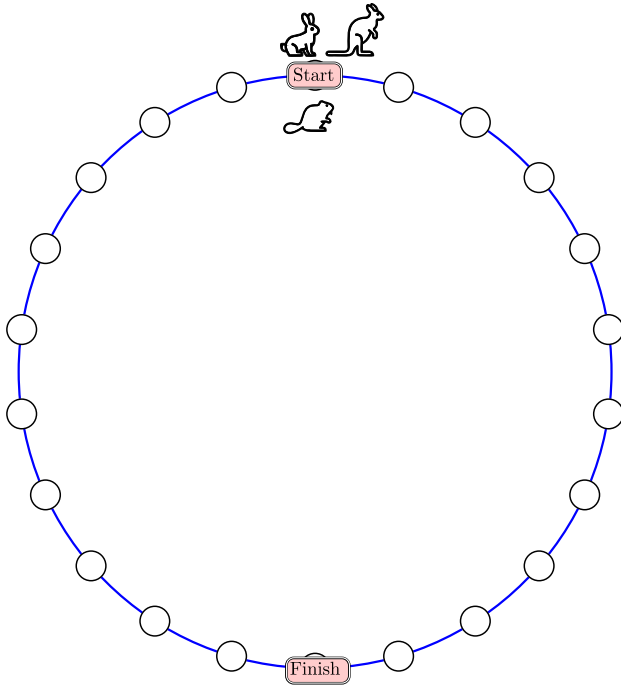
What is the smallest number of moves he needs to make to get to the correct order?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 6

SOLUTION: step 1: switch the top 2
step 2: switch all 4
step 3: switch top 3



18 (Switzerland). A rabbit, a beaver and a kangaroo are having a competition. The beaver moves one space at a time, the rabbit moves two spaces at a time and the kangaroo moves three spaces at a time. They all start from the point marked START. The winner is the animal who lands exactly on the point marked FINISH in the smallest number of complete moves. Who wins the competition?



- (A) the beaver (B) the rabbit (C) the kangaroo
(D) the kangaroo and the rabbit (E) the kangaroo and the beaver

SOLUTION: The beaver and kangaroo need both 11 jumps to reach the field FINISH. The rabbit will never reach the field FINISH at all.

19 (Greece). Lonneke wants the sum of the numbers in the white cells to equal the sum of the numbers in the grey cells. Which two numbers does she need to swap?

1	3	5	2	13
7	4	6	8	11

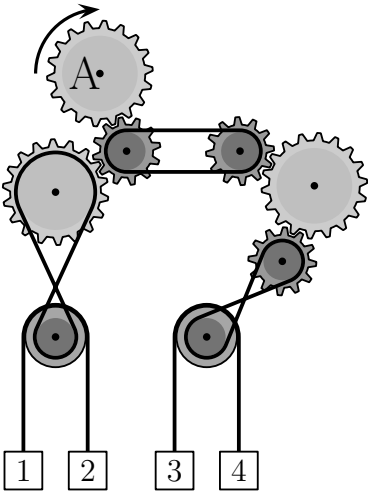
- (A) 1 and 11 (B) 2 and 8 (C) 3 and 7 (D) 4 and 13 (E) 7 and 13

SOLUTION: We do not work randomly or by trial and error. Instead we do some calculation: At the moment the sum of the grey cells is $1+2+7+4+6 = 20$ and of the white cells is $3+5+13+8+11=40$. If we want them the same, each sum must be 30. So we must increase the grey sum by 10 and at the same time decrease the white sum by 10. The only grey number which is 10 less than a white number is the pair 1 and 11 (it is easy to spot because 1 is the smallest grey number and 11 is the largest white, so no other difference of “white minus grey” is as large). The figure shows the numbers after

11	3	5	2	13
7	4	6	8	1

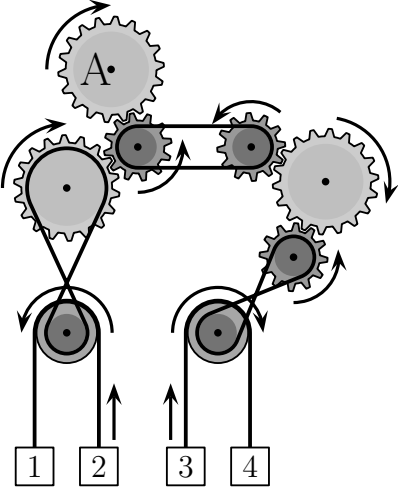
the swap.

20 (China). The gear marked A is turned clockwise, as shown. Which two boxes will move upwards?



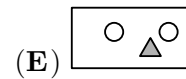
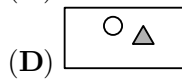
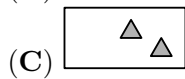
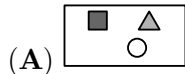
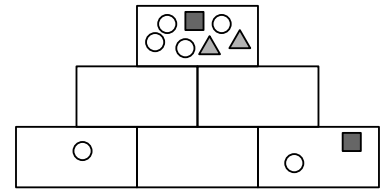
- (A) 1 and 4 (B) 2 and 3 (C) 1 and 3 (D) 2 and 4
(E) It cannot be determined

SOLUTION: Starting wheel A clockwise
Left-hand side — right-hand side
counter clockwise — counter clockwise
clockwise — clockwise
counter clockwise — counter clockwise
(done) — clockwise
Box 2 is lifted — Box 3 is lifted

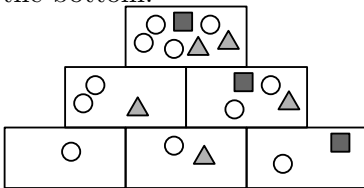


5 points

21 (Germany). Tian wants to draw figures in the six boxes of the pyramid shown. Each box should contain all of the figures in the two boxes directly below it and nothing more. She has drawn the figures in some of the boxes already. Which figures should she draw in the box in the middle of the bottom row?

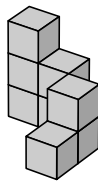


SOLUTION: In the middle row on the left is one circle and the figures from the middle box at the bottom. In the middle row on the right is one circle, one square and the figures from the middle box at the bottom. Hence, in the box at the top are two circles, one square and twice the figures in the middle box at the bottom. Therefore, there has to be one circle and one triangle in the middle box at the bottom.

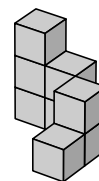
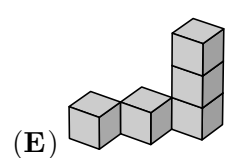
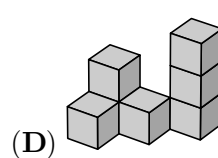
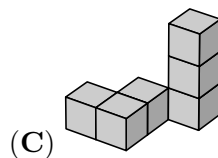
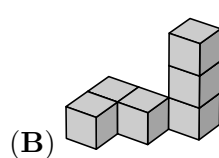
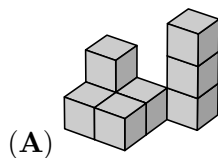


22 (Iran). Martha chose one of the five structures below and combined it with the structure on the right.

3	2	3
2	1	2
1	0	1



The table shows the number of cubes in each column in the combined structure when seen from above. Which of the five structures did Martha choose?



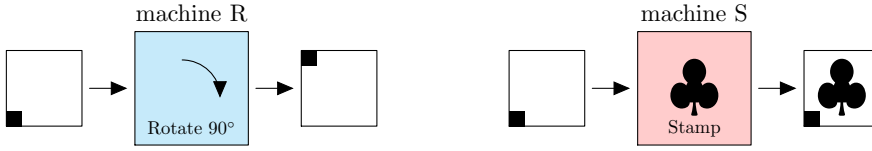
SOLUTION: The number of cubes in each column of the structure,

3	2	3
2	1	2
1	0	1

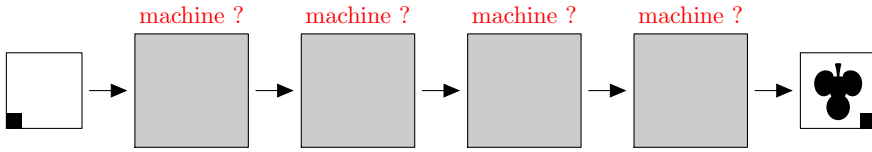
is shaded gray.

Hence Martha will have to choose a structure that fits with the non shaded numbers, and that is D. "Kangaroo" solution.: The sum of the numbers in the diagram is 15. The construction on the right has 8 cubes. So you need a structure made of 7 cubes - the only one is D. LM

23 (Denmark). Else has two machines. Machine R rotates the paper 90° clockwise. Machine S stamps the paper with a ♣.



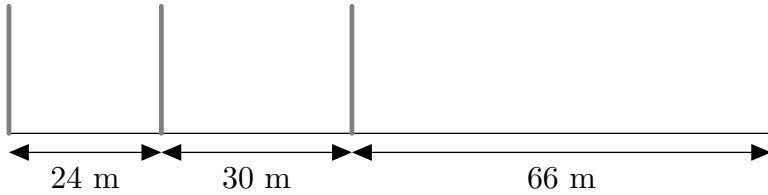
In which order are the machines used to create the image shown?



- (A) SRRR (B) RSRR (C) SRSR (D) RRRS (E) SRRS

SOLUTION: The paper has been turned 90 degrees clockwise three times, but only two time after it was stamped. Hence RSRR.

24 (Poland). Four stakes are placed along a 120m track, as shown.



What is the smallest number of stakes that should be added so that the track is divided into sections of equal length?

- (A) 12 (B) 15 (C) 17 (D) 20 (E) 37

SOLUTION: The greatest common divisor of 24, 30, 66 is 6. Thus one needs $120 : 6 + 1 = 21$ stakes; $21 - 4 = 17$.

25 (Poland). On a table there is a tower made of blocks numbered from 1 to 50. Emma builds a new tower in the following way. She takes two blocks from the top of the original tower and puts them on the table as the base of the new tower. She continues by taking the two top blocks from the remainder of the original tower and putting them on the top of the new tower, as seen in the diagram. Which of the following pairs of numbers are on adjacent blocks in the new tower?

50	2
49	1
4	48
3	47
2	50
1	49

- (A) 29 and 28 (B) 34 and 35 (C) 29 and 26 (D) 31 and 33 (E) 27 and 30

SOLUTION: In the new tower, below each block with an even number n there is a block with the number $n - 1$. Above each block with an even number n there is a block with the number $n - 3$. This condition is met only by the answer E.

26 (Norway). Martin has three cards with numbers written on both sides. The card with number 1 on one side has number 4 on the opposite side, the card with 2 on has 5 on the opposite side and the card with 3 on has 6 on the opposite side.

Martin randomly places three cards on the table and adds up the three numbers he sees.

How many different sums can Martin get?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 10

	Front	Back
Card 1	1	4
Card 2	2	5
Card 3	3	6

SOLUTION: The smallest sum is the sum of the numbers at the front of the cards. Every time you turn one of the cards, you increase the sum by 3. This can be done three times, hence there are four different sums.

27 (Iran). In a second hand shop, two hats are sold for the same price as five skirts, three skirts for the same price as eight t-shirts and two t-shirts for the same price as three caps. Which of the following collections is the most valuable?

- (A) a hat and five skirts (B) a hat, three skirts and a cap
(C) eight skirts and six t-shirts (D) thirty-seven caps
(E) three skirts and three caps

SOLUTION: We calculate the value compared to that of the cap: 2 t-shirts = 3 caps

3 skirts = 8 t-shirts = 12 caps, hence 1 skirt = 4 caps

2 hats = 5 skirts = 20 caps, hence 1 hat = 10 caps.

Now we determine the value of each options:

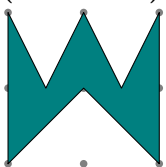
- a) $1 \cdot 10 + 5 \cdot 4 = 30$ caps
b) $1 \cdot 10 + 3 \cdot 4 + 1 = 23$ caps
c) $8 \cdot 4 + 6 \cdot \frac{3}{2} = 41$ caps
d) 37 caps
e) $3 \cdot 4 + 3 = 15$ caps

28 (Catalonia). Sonia and Robert are playing a game. They can alternately take 1, 2, 3, 4 or 5 tiles from a pile of tiles. Whoever takes the last tile or tiles loses. At one point of the game, there are 10 tiles left in the pile and it is Sonia's turn to take some tiles. How many tiles should Sonia leave to Robert to be sure that she will win?

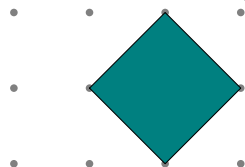
- (A) 9 (B) 8 (C) 7 (D) 6 (E) 5

SOLUTION: If a player is left with one tile, this player will lose. If a player is left with 2, 3, 4, 5 or 6 tiles, this player can leave 1 tile for the opponent, and will win. If a player is left with 7 tiles, the only option is to leave 2, 3, 4, 5 or 6 tiles for the opponent, hence this player will lose. If a player is left with 8 or 9 tiles, this player can leave 7 tiles for the opponent, and will win. Therefore Sonia should leave 7 tiles for Robert.

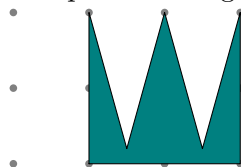
29 (Denmark). Which of the following four shapes has the greatest area?



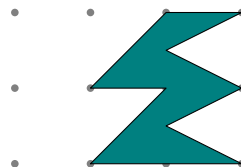
W



diamond



the crown

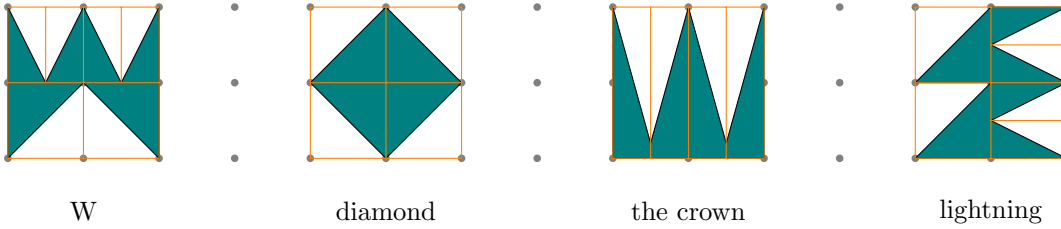


lightning

- (A) W (B) diamond (C) the crown (D) lightning

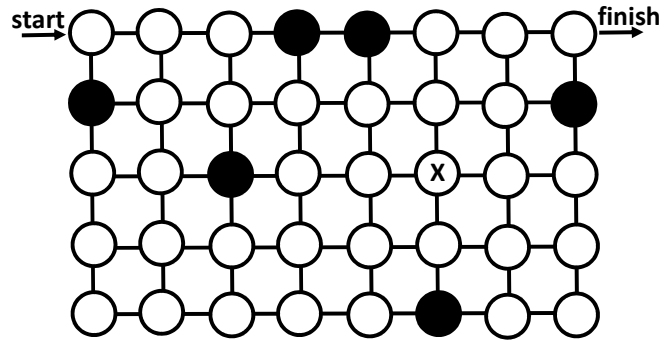
(E) they all have the same area

SOLUTION: The picture shows that the areas of W, the diamond and the lightning are exactly half of a 2×2 square, while the area of the crown is greater than half of a 2×2 square.



30 (Greece). An explorer wants to find a path through the maze shown from the point marked 'start' to the point marked 'finish'. She can only move horizontally or vertically and she can only pass through white circles. She also has to pass through all the white circles exactly once. When she reaches the circle marked X, what will her next move be?

- (A) $\uparrow$ (B) $\downarrow$ (C) $\rightarrow$ (D) $\leftarrow$
 (E) there is no such path



SOLUTION: We do not draw the path by random trial and error nor do we draw it at a stroke from start to finish. Rather, we draw it in parts after some thoughts. The thing to observe that some circles are connected with exactly one or exactly two others (shown in green in the figure). The path through those circles is without a choice, so we draw the corresponding part of the path (red parts in the figure). When we are done we will discover some circles (yellow in the figure) that are now connected with exactly two others because some of their neighbours are already in the path. On the basis of the yellow circles (not all are drawn here but we leave them to the reader) more parts of path can be added (not done here). When this is done we will have a sufficiently large portion of the path and it is easy to complete it, as shown (unique solution).

