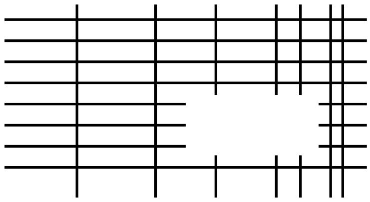
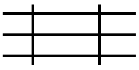
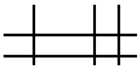
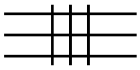
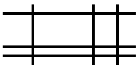
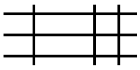
Cadet

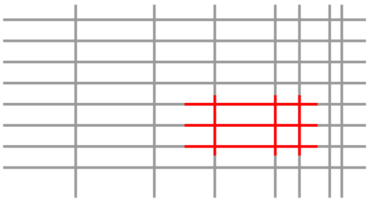
3 points

1 (Croatia). The diagram shows a set of horizontal and vertical lines with one part removed. Which of the following could be the missing part?

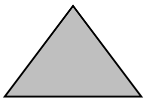


(**A**) 
(**B**) 
(**C**) 

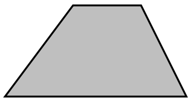
(**D**) 
(**E**) 

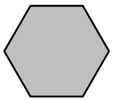
SOLUTION: 

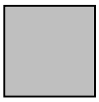
2 (Georgia). Which of the shapes below cannot be divided into two trapezia by a single straight line?

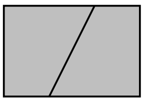
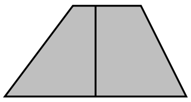
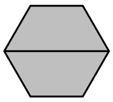
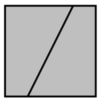

(**A**) triangle


(**B**) rectangle

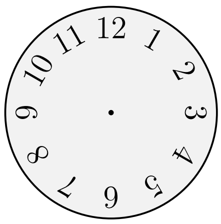

(**C**) trapezium

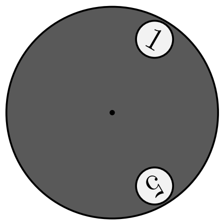

(**D**) regular hexagon


(**E**) square

SOLUTION: (B)  (C)  (D)  (E) 

3 (Denmark). A grey circle with two holes in it is placed on top of a clock-face, as shown.
The grey circle is turned around its centre such that an 8 appears in one hole. Which two numbers could be seen in the other hole?

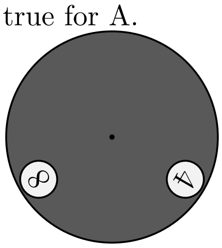


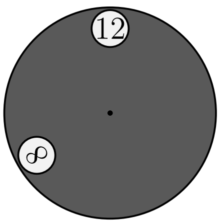


(**A**) 4 or 12 (**B**) 1 or 5 (**C**) 1 or 4

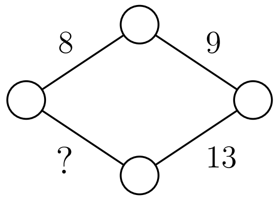
(**D**) 7 or 11 (**E**) 5 or 12

SOLUTION: The picture shows that one hole will show a time 4 hours after the other. This is only true for A.



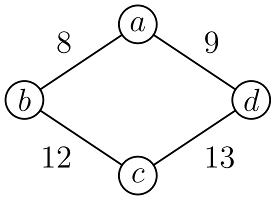


4 (Greece). Werner wants to write a number at each vertex and on each edge of the rhombus shown. He wants the sum of the numbers at the two vertices at the ends of each edge to be equal to the number written on the edge. What number will he write instead of the question mark?

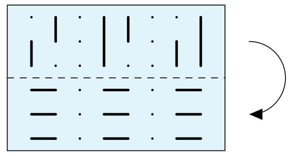


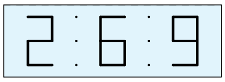
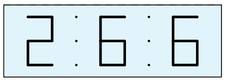
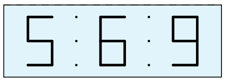
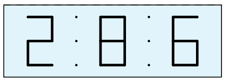
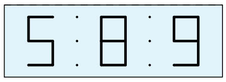
(**A**) 11 (**B**) 12 (**C**) 13 (**D**) 14 (**E**) 15

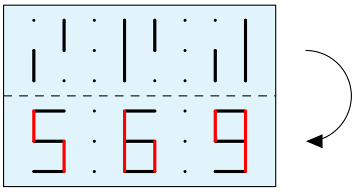
SOLUTION: Observe that the sum of the two numbers in any two opposite edges is the same. For example the edges with end points a, b and c, d, respectively, have sum $(a+b)+(c+d) = a+b+c+d$ which is the same as the sum $(a+d)+(b+c)$ of two other opposite edges. So in our case the number at the edge with the question mark is $8+13-9 = 12$.



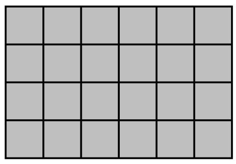
5 (Denmark). Kristina has a piece of transparent paper with some lines marked on it.
She folds it along the dashed line.
What can she now see?

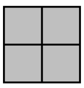
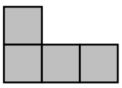
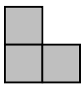
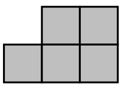
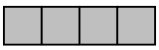


(**A**) 
(**B**) 
(**C**) 
(**D**) 
(**E**) 

SOLUTION: 

6 (Greece). A tiler wants to tile a floor of dimensions 4 m $\times$ 6 m using identical tiles. No overlaps or gaps are allowed. Which of the following tiles could not be used?



(**A**) 
(**B**) 
(**C**) 
(**D**) 
(**E**) 

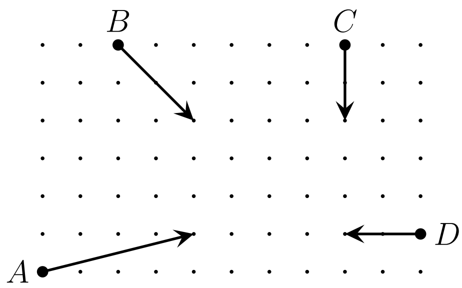
SOLUTION: As $4 \times 6 = 24$ is not a multiple of 5, tile D cannot be used because it has 5 squares. It is easy to see that all the other tiles can cover the floor.

7 (Australia). John has 150 coins. When he throws them on the table, 40% of them show heads and 60% of them show tails. How many coins showing tails does he need to turn over to have the same number show heads as show tails?

(**A**) 10 (**B**) 15 (**C**) 20 (**D**) 25 (**E**) 30

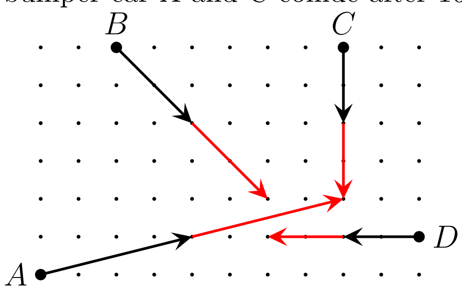
SOLUTION: He needs to turn over 10% of the coins which is 15 coins.

8 (Denmark). The diagram shows the initial position, the direction of travel and how far four bumper cars move in five seconds.
Which two cars will collide?

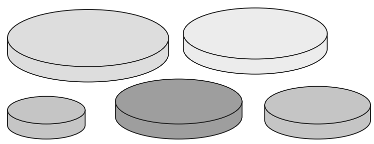


(**A**) A and B (**B**) A and C (**C**) A and D (**D**) B and C (**E**) C and D

Solution: By drawing the arrows showing the movement for 5 seconds more, it is easy to see that bumper car A and C collide after 10 seconds. No other pair will collide.



9 (Slovenia). Anna has five circular discs, each of a different size. She decides to build a tower using three of her discs so that each disc in her tower is smaller than the disc below it. How many different towers could Anna construct?



(**A**) 5 (**B**) 6 (**C**) 8 (**D**) 10 (**E**) 15

Solution: Name every disc considering its radius. Let D1 be the disc with the smallest radius and let D5 be the disc with the biggest radius. Denote (x,y,z) as the tower where x is the disc at the top of the tower, y is the disc in the middle and z is the disc at the bottom. it is easy to see that all possible towers are: (D3,D4,D5), (D2,D4,D5), (D1,D4,D5), (D2,D3,D5), (D1,D3,D5), (D1,D2,D5), (D2,D3,D4), (D1,D3,D4), (D1,D2,D4), (D1,D2,D3). This makes 10 possible towers in total.

10 (Mexico). Evita wants to write the numbers 1 to 8 in the boxes of the grid shown, so that the sums of the numbers in the boxes in each row are equal and the sums of the numbers in the boxes in each column are equal. She has already written numbers 3, 4 and 8, as shown. What number will she write in the shaded box?

	4		
3		8	

(**A**) 1 (**B**) 2 (**C**) 5 (**D**) 6 (**E**) 7

Solution: The total sum of all number 1 to 8 is 36, so the sum of the numbers in each row has to be 18 and the sum of the numbers in each column has to be 9. So the solution is:

6	4	1	7
3	5	8	2

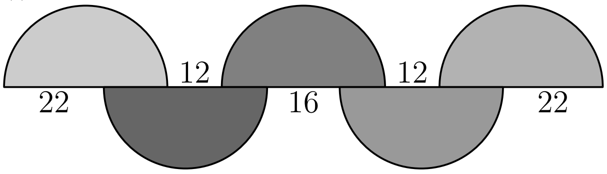
4 points

11 (Germany). Theodorika wrote down three consecutive whole numbers in order, but instead of digits she used symbols so wrote $\Box\Diamond\Diamond$, $\heartsuit\triangle\triangle$, $\heartsuit\triangle\Box$.
What would she write next?

(**A**) $\heartsuit\heartsuit\Diamond$ (**B**) $\Box\heartsuit\Box$ (**C**) $\heartsuit\triangle\Diamond$ (**D**) $\heartsuit\Diamond\Box$ (**E**) $\heartsuit\triangle\heartsuit$.

Solution: From the change of the first digit we conclude that $\Diamond = 9, \triangle = 0, \Box = 1$. So the first number is 199, and we have $\heartsuit = 2$. So, the next number to write is 202, that means $\heartsuit\triangle\heartsuit$.

12 (Iran). The diagram shows five equal semicircles and the lengths of some line segments. What is the radius of the semicircles?



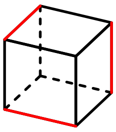
(**A**) 12 (**B**) 16 (**C**) 18 (**D**) 22 (**E**) 36

Solution: Let r be the radius of the circles. It is clear that $6r + 12 + 12 = 4r + 22 + 16 + 22$ so $r = 18$.

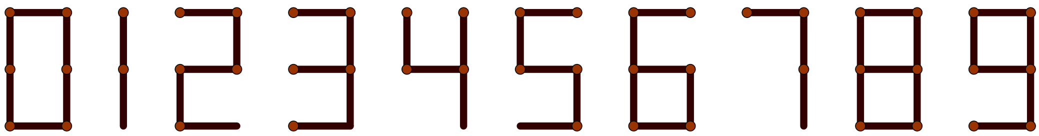
13 (Belarus). Some edges of a cube are to be coloured red so that every face of the cube has at least one red edge. What is the smallest possible number of edges that could be coloured red?

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

Solution: It is clear that just 2 edges coloured red is not enough whether a configuration with 3 edges coloured is possible. In the diagram shown there is an example of such a colouring.



14 (Austria). Matchsticks can be used to write digits, as shown in the diagram.



How many different positive integers can be written using exactly six matchsticks in this way?

(**A**) 2 (**B**) 4 (**C**) 6 (**D**) 8 (**E**) 9

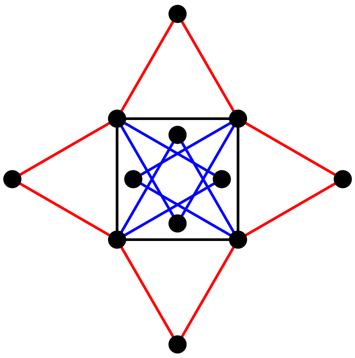
Solution: First of all, we count the number of matchsticks required to write each of the digits. If each pair (x, y) has x as the digit and y as the number of matchsticks required to write it, we have the pairs

$$(0, 6), (1, 2), (2, 5), (3, 5), (4, 4), (5, 5), (6, 6), (7, 3), (8, 7), (9, 6).$$

We note $6 = 2+4 = 4+2 = 3+3 = 2+2+2$. No digit is written with a single matchstick. The digit 4 requires 4 matchsticks, the digit 1 requires 2 matchsticks and the digit 7 requires 3 matchsticks. This gives us the six possibilities: $6, 9, 14, 41, 77, 111$.

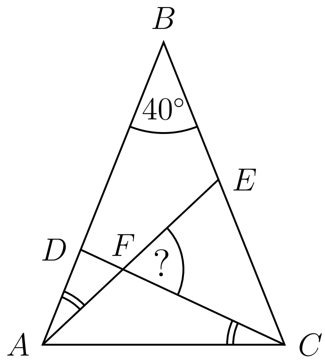
15 (Hungary). The edges of a square are 1 cm long. How many points on the plane are exactly 1 cm away from two of the vertices of this square?

(**A**) 4 (**B**) 6 (**C**) 8 (**D**) 10 (**E**) 12



SOLUTION:

16 (Catalonia). Triangle ABC is isosceles with $\angle ABC = 40°$. The two marked angles, $\angle EAB$ and $\angle DCA$, are equal. What is the size of the angle $\angle CFE$?



(**A**) 55°

(**B**) 60°

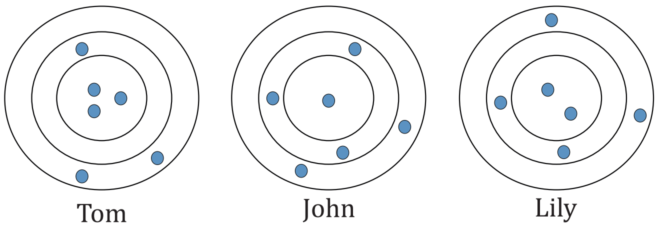
(**C**) 65°

(**D**) 70°

(**E**) 75°

SOLUTION: First we observe that $\angle BCA = \angle CAB = \dfrac{180° - 40°}{2} = 70°$. In $\triangle FAC$ it is clear that $\angle AFC = 180° - \angle FCA - (70° - \angle FCA) = 110°$. So, $\angle ? = 180° - 110°$.

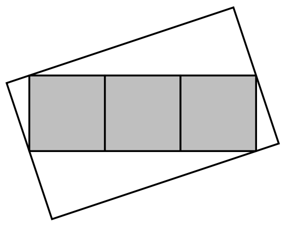
17 (China). Tom, John and Lily each shot six arrows at a target. Arrows hitting anywhere within the same ring score the same number of points. Tom scored 46 points and John scored 34 points, as shown. How many points did Lily score?



(**A**) 37 (**B**) 38 (**C**) 39 (**D**) 40 (**E**) 41

SOLUTION: Observe that if we add the numbers of hits in each ring from Tom and John together they hit each ring exact the double amount of times of the number of hits of Lily. Since the sum of the points of Tom and John is $46 + 34 = 80$, Lily scored 40 points.

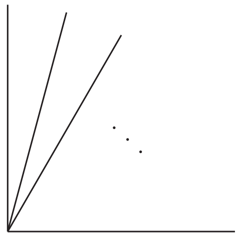
18 (Poland). The diagram shows a rectangle made from three grey squares, each of area 25 cm^2, inside a larger white rectangle. Two of the vertices of the grey rectangle touch the mid-points of the shorter sides of the white rectangle and the other two vertices of the grey rectangle touch the other two sides of the white rectangle.
What is the area, in cm^2, of the white rectangle?



(**A**) 125 (**B**) 136 (**C**) 149 (**D**) 150 (**E**) 172

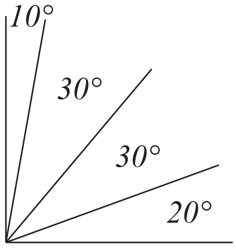
SOLUTION: The big white right triangle at the rectangle's lower vertex can be shifted up to the other big white right triangle so that their hypotenuses match. Likewise the small white right triangle at the rectangle's vertex to the right can be shifted left to the other small triangle. Now we can see that the white area is exactly half of the rectangle's area, hence the three squares together with area 75 cm^2 are the other half of the rectangles area. So the area of the white rectangle is $2 \cdot 75$ cm^2 $= 150$ cm^2.

19 (Belarus). Angelo drew two lines meeting at a right-angle. What is the smallest number of extra lines he could draw inside his right-angle, as shown, so that for any of the values $10°$, $20°$, $30°$, $40°$, $50°$, $60°$, $70°$ and $80°$, a pair of lines can be chosen with the angle between them equal to that value?



(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

SOLUTION: It's clear that two extra lines is not enough. Example for three extra lines:



20 (Serbia). The sum of 2023 consecutive integers is 2023. What is the sum of digits of the largest of these integers?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

SOLUTION: Using only positive consecutive integers, i. e. 1..2023, their sum is way too big, so the sequence must start with negative numbers, such that many of them cancel out. Equally balanced $-1011..1011$ yields 0 as sum. Just shifting this block one over leads to $-1010..1012$, where $-1010..1010$ cancel out and $1011, 1012$ with sum 2023 are left over and $1+0+1+2=4$.

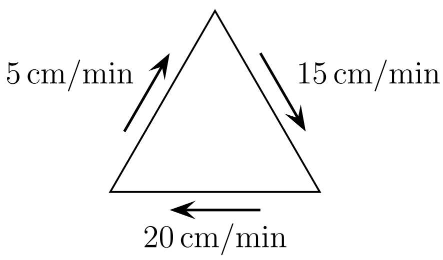
5 points

21 (Puerto Rico). Some beavers and some kangaroos are standing in a circle. There are three beavers in total and there are no two beavers who are standing next to another beaver. There are exactly three kangaroos who are standing next to another kangaroo. What is the largest possible amount of kangaroos in the circle?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

SOLUTION: On both sides of the exactly three kangaroos K standing next to one another there must be beavers B, so the sequence BKKKB is already given. As no two beavers are standing next to another, kangaroos must follow on both sides of this sequence giving KBKKKBK and the only remaining beaver must close the circle. Hence 5 kangaroos is the maximum.

22 (China). An ant is walking along the sides of an equilateral triangle. The speeds at which it travels along the three sides are 5 cm/min, 15 cm/min and 20 cm/min, as shown. What is the average speed, in cm/min, at which the ant walks the whole perimeter of the triangle?



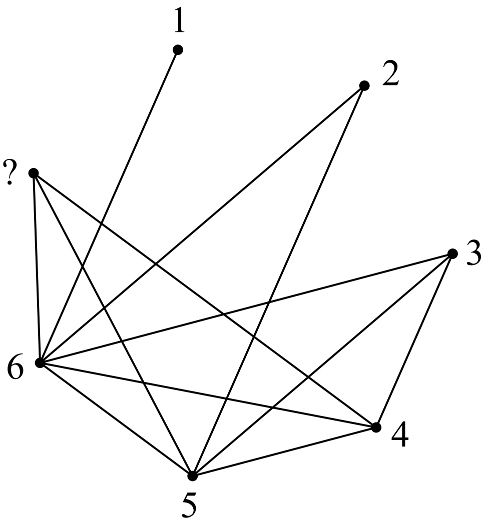
(A) 10 **(B)** $\frac{80}{11}$ **(C)** $\frac{180}{19}$ **(D)** 15 **(E)** $\frac{40}{3}$

SOLUTION: Assume each side of the triangle to be x cm. Total time taken $= \frac{x}{5}$ min$+\frac{x}{15}$ min$+\frac{x}{20}$ min $= \frac{19x}{60}$ min. Total distance $= 3 \times x$ cm. Average speed $= \dfrac{3x \text{ cm}}{\frac{19x}{60} \text{ min}} = \frac{180}{19}$ cm/min.

23 (Hungary). Snow White organised a chess competition for the seven dwarves, in which each dwarf played one game with every other dwarf. On Monday, Grumpy played 1 game, Sneezy played 2, Sleepy 3, Bashful 4, Happy 5 and Doc played 6 games. How many games did Dopey play on Monday?

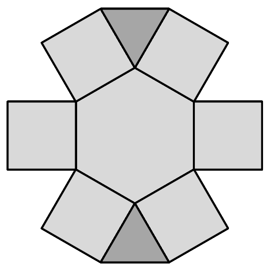
(**A**) 1 (**B**) 2 (**C**) 3 (**D**) 4 (**E**) 5

Solution: You can draw the matches on Monday.



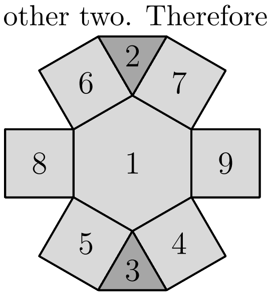
You can see, that Dopey played 3 matches.

24 (Iran). Elizabetta wants to write the numbers 1 to 9 in the regions of the shape shown so that the product of the numbers in any two adjacent regions is not more than 15. Two regions are said to be adjacent if they have a common edge. In how many ways can she do this?



(**A**) 12 (**B**) 8 (**C**) 32
(**D**) 24 (**E**) 16

Solution: Note that 8 and 9 must have at most 1 neighbor and that neighbor must also be 1. As a result, the numbers 8 and 9 must be in areas 8 and 9, which can be in two ways. Also, the number can only be 1 in area 1. Now we specify the position of the number 7, which has four states because it cannot be in areas 2 and 3, and the triangle adjacent to the number 7 can only be the number 2. Since the number 6 cannot be adjacent to the number 3, it will come next to the triangle with the number 2. Now we will have two modes for placing the numbers 4 and 5, only 3 can be adjacent to the other two. Therefore, there are $2 \times 4 \times 2$ ways to place the numbers 1 to 9 with the given conditions.

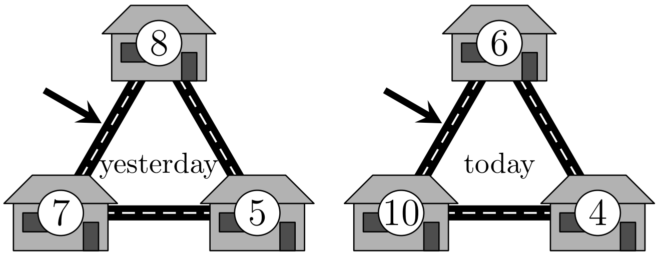


25 (Catalonia). Martin is standing in a queue. The number of people in the queue is a multiple of 3. He notices that he has as many people in front of him as behind him. He sees two friends, both standing behind him in the queue, one in 19th place and the other in 28th place. In which position in the queue is Martin?

(**A**) 14 (**B**) 15 (**C**) 16 (**D**) 17 (**E**) 18

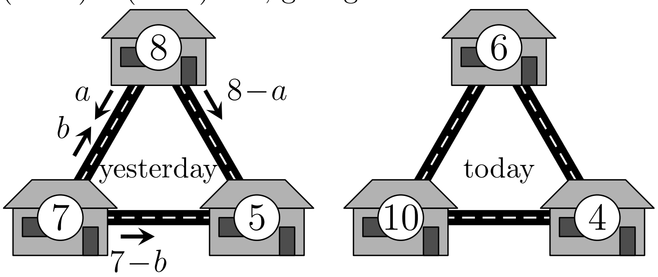
Solution: Let's say Martin has n people in front and n behind him, that means Martin is in position $n+1$. On the other hand $n+1 < 19$ and $2n+1 > 28$, that is n can only be 14, 15, 16 or 17. Since $2n+1$ must be a multiple of 3, $n = 16$ and Martin is in the 17th position.

26 (Greece). Some mice live in three neighbouring houses. Last night, every mouse left its house and moved to one or the other of the other two houses, always taking the shortest route. The numbers in the diagram show the number of mice per house, yesterday and today. How many mice used the path shown by the arrow?



(**A**) 9 (**B**) 11 (**C**) 12 (**D**) 16 (**E**) 19

Solution: First method: The number of mice the left the two houses on the left is $7+8=15$. Some or none of these mice crossed the path shown (connecting each other's house) but the rest went to the third house. We see form the second picture that 4 mice arrived at this third house. It means that the other ones, namely $15-4=11$, are the mice that crossed the shown path. Second solution: Suppose the number of mice that left the two houses on the left to go to each other are a and b, respectively. We want to find $a+b$. Clearly the number of mice that went to the third house are $8-a$ and $7-b$, respectively. From the second picture we know that 4 mice arrived at the third house, so $(8-a)+(7-b)=4$, giving $a+b=8+7-4=11$.



27 (Greece). Bart wrote the number 1015 as a sum of numbers using only the digit 7. He used a 7 a total of 10 times, as shown. Now he wants to write the number 2023 as a sum of numbers using only the digit 7, using a 7 a total of 19 times. How many times will he use the number 77?

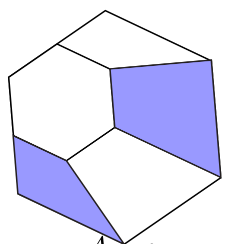
$$\begin{array}{r} 777 \\ 77 \\ +\ 77 \\ 77 \\ \underline{7} \\ 1015 \end{array}$$

(**A**) 2 (**B**) 3 (**C**) 4 (**D**) 5 (**E**) 6

Solution: Solution. Since the units digit of 2023 is a 3, we must use a column of nine 7's because the only multiple of 7 ending in a 3 is $7 \times 9 = 63$. This gives 6 to carry. So the tens column must end on a 6 (so that 6+6 ends on a 2), which means we need a column of eight 7's as $8 \times 7 = 56$. Now this column has a 6 to carry so we need 14 =7+7 on the left most column. Alternative solution: Three 777's are too much as $3 \times 777 = 2331 > 2023$ and one 777 is too little. So start with two 777's. What remains is $2023 - 2 \times 777 = 469$, which is easy to manage. (Additional comment: We can also write 2023 using 37 sevens as $777 + 16 \times 77 + 2 \times 7$. Here the units column has 19 elements).

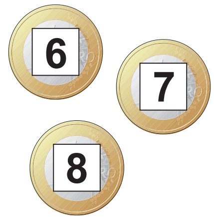
$$\begin{array}{r} 777 \\ 777 \\ 77 \\ 77 \\ +\ 77 \\ 77 \\ 77 \\ 77 \\ \underline{7} \\ 2023 \end{array}$$

28 (Brazil). A regular hexagon is divided in four quadrilaterals and one smaller regular hexagon. The area of the shaded region and the area of the small hexagon are in the ratio $\frac{4}{3}$. What is the ratio $\frac{\text{area small hexagon}}{\text{area big hexagon}}$?



(**A**) $\frac{3}{11}$ (**B**) $\frac{1}{3}$ (**C**) $\frac{2}{3}$ (**D**) $\frac{3}{4}$ (**E**) $\frac{3}{5}$

Solution: A := area big hexagon, a := area small hexagon. Then shaded area $= \frac{A-a}{2}$ and $\frac{\text{shaded area}}{a} = \frac{4}{3}$ leads to $\frac{a}{A} = \frac{3}{11}$.



29 (Slovakia). Jake wrote six consecutive numbers onto six white pieces of paper, one number on each piece. He stuck these bits of paper onto the top and bottom of three coins. Then he tossed these three coins three times. On the first toss, he saw the numbers 6, 7 and 8, as shown, and then coloured them red. On the second toss, the sum of the numbers he saw was 23 and on the third toss the sum was 17. What was the sum of the numbers on the remaining three white pieces of paper?

(A) 18 **(B)** 19 **(C)** 23 **(D)** 24 **(E)** 30

SOLUTION: If those six numbers are consecutive and $6, 7, 8$ were seen, there are only four possibilities for all six numbers: 3..8, 4..9, 5..10, and 6..11. The sum 24 cannot be obtained with 3 numbers out of 3..8 and the sum 17 cannot be obtained with 3 numbers out of 5..10, and 6..11. So it must be the numbers 4..9 and the sum of the numbers on the three white pieces of paper must be $4 + 5 + 9 = 18$.

30 (South Africa). A rugby team scored 24 points, 17 points and 25 points in the seventh, eighth and ninth games of the 2022 season. Their average points-per-game was higher after 9 games than it was after their first 6 games. Their average after 10 games was more than 22. What is the smallest number of points that they could have scored in their 10th game?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

SOLUTION: The average in the seventh, eighth and ninth games was (24+17+25)/3=66/3=22. This means that the average of the first 6 games was less than 22. Over the first 6 games, the team could have scored 22x6-1=132-1=131 points. After 10 games, the average must be more than 22, so over the 10 matches the team must have scored at least 10x22+1=220+1=221 points in total. The smallest number that could have been scored in the last match is 221-131-66=24