1. At Stanford, happiness is measured in Stanjoys. While working on math homework without interruption, Lockensy earns Stanjoys at a continuous rate of $\frac{t^2}{7}$ Stanjoys per minute, where t is the number of minutes since she last resumed uninterrupted homework. While not doing her math homework, she earns Stanjoys at a constant rate of 1 per minute. During a 1-hour homework session, Lockensy takes an 18-minute phone call from her friend Chillsea. Given that Chillsea times her call to minimize Lockensy's total Stanjoys, compute the amount of Stanjoys that Lockensy will accumulate by the end of the hour session.

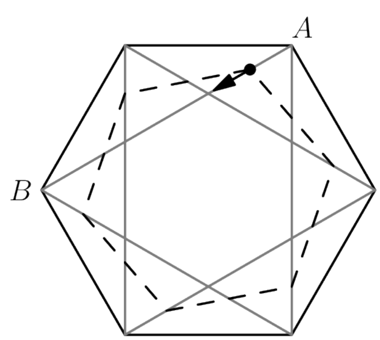
2. Let $f(x) = e^{\cos^2(x)}$ and $g(x) = e^{\sin^2(x)}$. Compute
$$\int_0^{\pi} f'(x)g'(x)\,\mathrm{d}x.$$

3. Given real constants a, b, c, define the cubic polynomials
$$A(x) = ax^3 + abx^2 - 4x - c$$
$$B(x) = bx^3 + bcx^2 - 6x - a$$
$$C(x) = cx^3 + cax^2 - 9x - b.$$
Given that A, B, and C have local extrema at b, c, and a, respectively, compute abc. (A local extremum is either a local maximum or a local minimum.)

4. Let f be a continuous function satisfying $f(x^7 + 6x^5 + 3x^3 + 1) = 9x + 5$ for all reals x. Compute the integral
$$\int_{-9}^{11} f(x)\,\mathrm{d}x.$$

5. Consider the set of all continuous and infinitely differentiable functions f with domain $[0, 2025]$ satisfying $f(0) = 0$, $f'(0) = 0$, $f'(2025) = 1$, and f'' being strictly increasing on the interval $[0, 2025]$. Compute the smallest real number M such that for all functions f in this set, $f(2025) < M$.

6. In the diagram, the larger regular hexagon has side length 1. A black dot moves with constant velocity from A to B, two vertices on the larger hexagon as shown in the diagram. The dashed regular hexagon is then drawn with one of its vertices on the black dot while remaining rotationally symmetric with respect to the center of the larger hexagon (i.e. the dashed hexagon has the same center as the large hexagon). Compute the average area of the dashed hexagon over time as the black dot travels from A to B.



7. Compute
$$\lim_{t\to 0}\left(\prod_{n=2}^{\infty}\left(1 + \frac{t}{n^2+n}\right)\right)^{\frac{1}{t}}.$$

8. Let R be the region in the complex plane enclosed by the curve $f(\theta) = e^{i\theta} + e^{2i\theta} + \frac{1}{3}e^{3i\theta}$ for $0 \leq \theta \leq 2\pi$. Compute the perimeter of R.

9. Consider the function

$$f(x) = \frac{(2025 + x)\ln(2025 + x)}{x^3 - 6x^2 + 11x - 6}$$

defined for all real numbers except $x = 1, 2,$ and 3. For a positive integer n, let d_n denote $\frac{f^{(n)}(0)}{n!}$, where $f^{(n)}(x)$ is the n^{th} derivative of $f(x)$. Compute $\lim_{n\to\infty} d_n$.

10. Let S be a sphere centered at the origin O with radius 1. Let $A = (0, 0, 1)$. Let B and C be two points (not equal to A or each other) on the sphere's surface with equal z coordinate. Let minor arc AB be the shorter distance on the intersection of the sphere with the plane containing A, B, and O and similarly, let minor arc AC be the shorter distance on the intersection of the sphere with the plane containing A, C, and O. Let the minor arc BC be the shorter distance on the intersection of the sphere with the plane parallel to the xy plane and at the same z value as B (or either if the two arcs have the same length). The three arcs combined divide the surface of the sphere into two regions. Given that the area of smaller region enclosed by arcs AB, AC, and BC is 3, compute the minimum possible sum of the arc lengths of AB, AC, and BC.

TB. *This is an estimation question used for tiebreaking purposes. Ties on this test will be broken by absolute distance from the correct answer to this question.*

Let

$$I = \int_{\pi}^{2025\pi} \frac{\sin^2(x)}{\ln x}\, dx.$$

Estimate I in the decimal form $abcdef.ghij$, where $a, b, c, d, e, f, g, h, i, j$ are decimal digits each between 0 and 9, inclusive (leading zeros are possible).